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A STUDY OF SCALE FREE OPTICS IN NANODISORDERED FERROELECTRICS

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ABSTRACT
Diffraction is a fundamental limit for optics. Diffraction of light can be compensated by an index of refraction pattern or because of a nonlinear index change or nonlinearity. Nonlinearity can introduce new spatial scales. If one is able to identify a nonlinearity that introduces intensity independent scale that cancels the wavelength, a propagation of scale –free optics can occur. The idea behind scale-free optics can be studied by considering paraxial ray scalar approximation. In this work, paraxial ray approximation of scale free optics has been considered which results a Gausian scale free solution

I. INTRODUCTION
There is a very limited number of ways for artificially changing the refractive index of medium. But if one could be able to arbitrary increase it, unprecedented possibilities would open the road to a novel generation of optical device. The basic idea underlying scale-free propagation is that when light propagates in a medium in which non-linearity introduces an intensity- independent response that amounts to anti-diffraction[1,5].

Recent experiments report on the demonstration of scale-free propagation in disordered ferroelectric KTN:Li (a newly engineered cu-doped lithium enriched potassium-tantalate-niobate) crystals[6]. In this system a specific role is played by i) ferro-electricity, ii) photo-refraction i) Ferro-electricity is the property of a dielectric to manifest, below a specific Curie temperature, a spontaneous breaking of the crystal symmetry and an associated finite static electric polarization that can be switched through an external bias field. ii) A ferroelectric electro-optic crystal can also be photorefractive. Photo refraction is characterized by a strong optical non-linearity mediated by an indirect optical self-action: light photo induces charges from in-band impurities that redistribute in the crystal through drift and diffusion and give rise to a space-charge field which, through the electro- optic effect, changes the index of refraction and hence the propagation of the light itself. To understand the phenomena of diffraction cancelation, we recall that photo-refraction leads to a diffusive nonlinearity[7, 8], which profoundly alters beam propagation, in that diffraction is governed by an effective refractive index[9,11] n_{eff}

$$n_{eff} = \frac{n_0}{(1-(L/\lambda)^2)}, \tag{1}$$

Where n_0 is the unperturbed refractive index, λ the wavelength and $L = 4\pi n_0^2 \varepsilon_0 \sqrt{g}\chi_{NPR}(K_B T/q)$. Here, g is the effective quadratic electro-optic coefficient, χ_{NPR} is the effective history-dependent low-frequency dielectric susceptibility of the dipolar glass, K_B the Blotzmann constant, T the crystal equilibrium temperature (i.e. the temperature measured at a given instant) and q is the charge of the photo-excited carriers. Eqn.(1) is valid for $\lambda \geq L$. As $L \to \lambda$, $n_{eff} \gg n_0$ and diffraction is cancelled, the scale-free regime, independently of beam size and intensity. Scale- free optics opens the way to a number of enticing effects, such as wavelength-intensive propagation[10] and scale-free spatial instability[12].

II. MODEL

In order to grasp the core idea behind scale-free optics, we consider the propagation of an optical wave in the paraxial scalar approximation[13]. The slowly varying part of the optical field A (i.e. $|A|^2 = I$ is the optical intensity) obeys the paraxial wave equation

$$2ik\partial_Z A + \nabla_\perp^2 A + \frac{2k^2}{n}\Delta n A = 0 \qquad (2)$$

Where, $k = (\omega/c)n$ is the wave number, ω is the optical angular frequency, z is the propagation direction of the beam and $\perp \equiv (x, y)$ are the two transverse coordinates and n is the electro optic response of the PNR [14-19]. It is expressed as

$$\Delta n_{PNR} = -\frac{n^3}{2} g\varepsilon_0^2 \chi_{PNR}^2 E \qquad (3)$$

and the diffusive photo-induced electric field is

$$E = -\frac{K_B T}{q}\frac{\Delta I}{I} \qquad (4)$$

Substituting (3) in (2)

$$\Delta n = \Delta n_{PNR} = -\frac{n^3}{2} g\varepsilon_0^2 \chi_{PNR}^2 \left(\frac{K_B T}{q}\right)^2 \frac{(\partial_x I)^2 + (\partial_y I)^2}{I^2} \qquad (5)$$

With $g = g_{11} + g_{12}$ (that depends on the specific PNR- supporting ferroelectric used).
Inserting the non linear response term, Δn in equation (2), the nonlinear propagation equation is

$$i\frac{\partial A}{\partial Z} + \frac{1}{2k}\nabla_\perp^2 A - K\frac{\left(\frac{\partial |A|^2}{\partial x}\right)^2 + \left(\frac{\partial |A|^2}{\partial y}\right)^2}{|A|^4} A = 0, \qquad (6)$$

Where

$$K = kg\left(\frac{n\varepsilon_0 \chi_{PNR} K_B T}{\sqrt{2}q}\right)^2$$

The focusing/defocusing nature of the effect depends on the sign of $g = g_{11} + g_{12}$ .i.e., on the specific lattice structure of the underlying composite crystal.

In the case of KTN:Li, here considered, $g_{11} \rangle 0$ is dominant with respect to $g_{12} \langle 0$ which is an order of magnitude smaller, such that $g_{11} + g_{12} \rangle 0$ (g_{11}=0.16 m ^{4} , g_{12}=-0.02 m ^{4} and the effect is self focusing[14].
Introducing the anstaz, $A(x, y, z) = A_0(x, y, z)e^{-i\Omega(x,y,z)}$ in equ. (6)

$$\left(i\frac{\partial A_0}{\partial Z}+A_0\frac{\partial\Omega}{\partial Z}\right)$$

$$+\frac{1}{2k}\left\{\left(\frac{\partial^2 A_0}{\partial x^2}+\frac{\partial^2 A_0}{\partial y^2}\right)-2i\left(\frac{\partial A_0}{\partial x}\frac{\partial\Omega}{\partial x}+\frac{\partial A_0}{\partial y}\frac{\partial\Omega}{\partial y}\right)-i\left(A_0\frac{\partial^2\Omega}{\partial x^2}+A_0\frac{\partial^2\Omega}{\partial y^2}\right)-\left(A_0\left(\frac{\partial\Omega}{\partial x}\right)^2+A_0\left(\frac{\partial\Omega}{\partial x}\right)^2\right)\right\}$$

$$-K\frac{\left(\frac{\partial|A_0|^2}{\partial x}\right)^2+\left(\frac{\partial|A_0|^2}{\partial y}\right)^2}{|A_0|^4}A_0=0$$

(7)

Equating real and imaginary parts separately, we get following two equations:

$$A_0\frac{\partial\Omega}{\partial Z}+\frac{1}{2k}\left\{\left(\frac{\partial^2 A_0}{\partial x^2}+\frac{\partial^2 A_0}{\partial y^2}\right)-\left(A_0\left(\frac{\partial\Omega}{\partial x}\right)^2+A_0\left(\frac{\partial\Omega}{\partial y}\right)^2\right)\right\}-K\frac{\left(\frac{\partial|A_0|^2}{\partial x}\right)^2+\left(\frac{\partial|A_0|^2}{\partial y}\right)^2}{|A_0|^4}A_0=0$$

(8)

and

$$\frac{\partial A_0}{\partial Z}-\frac{1}{k}\left(\frac{\partial A_0}{\partial x}\frac{\partial\Omega}{\partial x}+\frac{\partial A_0}{\partial y}\frac{\partial\Omega}{\partial y}\right)-\frac{1}{2k}\left(A_0\frac{\partial^2\Omega}{\partial x^2}+A_0\frac{\partial^2\Omega}{\partial x^2}\right)=0 \qquad (9)$$

We look for a solution of the form

$$A_0(x,y,z)=\frac{A_{00}}{\sqrt{f_1(z)}\sqrt{f_2(z)}}e^{-\frac{x^2}{2r_0^2f_1^2(z)}}e^{-\frac{y^2}{2r_0^2f_2^2(z)}} \qquad 10(a)$$

and

$$\Omega=\frac{x^2}{2}\beta_1(z)+\frac{y^2}{2}\beta_2(z) \qquad 10(b)$$

Where, $\beta_1(z)=-\frac{k}{f_1}\frac{\partial f_1}{\partial Z}$ and $\beta_2(z)=-\frac{k}{f_2}\frac{\partial f_2}{\partial Z}$ and r_0f_1 and r_0f_2 are the beam width parameters in the x and y directions respectively f_1 and f_2 are functions of z. Putting 10(a) and 10(b) in equ.(8) we get

$$\frac{kx^2}{2f_1}\frac{\partial^2 f_1}{\partial Z^2}+\frac{ky^2}{2f_2}\frac{\partial^2 f_2}{\partial Z^2}+K\frac{4x^2}{f_1^4r_0^4}+K\frac{4y^2}{f_2^4r_0^4}-\frac{x^2}{2kf_1^4r_0^4}-\frac{y^2}{2kf_2^4r_0^4}+\frac{1}{2kf_1^2r_0^2}+\frac{1}{2kf_2^2r_0^2}=0$$

(11)

Collecting the coefficients of and from eqn. (11), we can easily derive the following pair of coupled nonlinear differential equations.

$$\frac{\partial^2 f_1}{\partial Z^2} = \left(\frac{1}{k^2}\right)\frac{1}{f_1^3 r_0^4} - \left(\frac{8K}{k}\right)\frac{1}{f_1^3 r_0^4} \qquad 12(a)$$

$$\frac{\partial^2 f_2}{\partial Z^2} = \left(\frac{1}{k^2}\right)\frac{1}{f_2^3 r_0^4} - \left(\frac{8K}{k}\right)\frac{1}{f_2^3 r_0^4} \qquad 12(b)$$

For $f_1 = f_2 = 1$

$$\frac{1}{k^2 r_0^4} - \frac{8K}{k r_0^4} = 0$$

$\frac{1}{k} - 8K = 0$ or $8K = \frac{1}{k}$

Therefore, $8Kk = 1$ (13)

Eq.(13) can be stated in terms of PNR susceptibility: $\chi_{PNR} \geq \chi_{thr} \simeq 10^5$, which also states that there exist a critical value for the non-linear optical response due the PNR[15-19] for which $L = \lambda$ ($\lambda = 632.8nm$ in our experiments). Notably enough, the density and the size of the PNR, that are directly related to the cooling rate, determine χ_{PNR} : as a result the scale-free regime will exist only above a cooling rate threshold. Diffraction free (zero effective wavelength) solutions of Eq.(6) are scale free. This effect is found in the Gaussian exact solution for $8kK = 1$.

III. CONCLUSION

We find that the Gaussian scale-free solutions when the condition $8kK = \frac{L^2}{\lambda^2} = 1$ is satisfied and diffraction is fully cancelled. When, $8kK > 1$, a wholly new optics can be predicted.

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